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# USE OF THE PUSHOVER METHOD FOR THE SEISMIC ANALYSIS OF MIXED FRAME-WALL STRUCTURES

**SPEAKER: GIOVANNI DI SCIASCIO, ENG.**



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## REQUIREMENTS

The study of the seismic behavior of buildings with a mixed frame-wall structure places the designer in front of important conceptual problems, such as the verification of the connections between the different construction elements, as well as the distribution of horizontal actions among the resisting walls and the torsional deformability due to the eccentricity between center of stiffness and center of mass.



## REQUIREMENTS

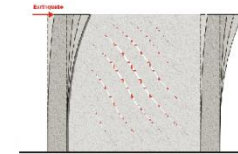
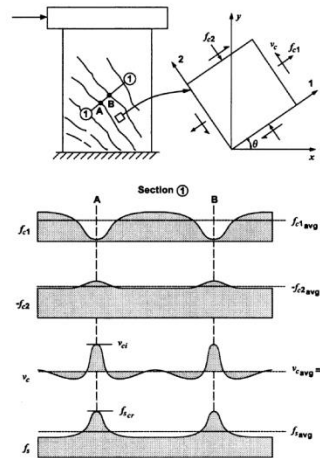
The proposed solution consists in implementing the suite of calculation (Straus7) with an Add-on that executes nonlinear static analysis (Pushover analysis) for reinforced concrete buildings, with the following features:



## REQUIREMENTS

The proposed solution consists in implementing the suite of calculation (Straus7) with an Add-on that executes nonlinear static analysis (Pushover analysis) for reinforced concrete buildings, with the following features:

- Calculation of nonlinear reinforced concrete slabs and walls modeled with Plate/Shell elements according to the Modified Compression Field Theory (MCFT) for slab elements and the Disturbed Stress Field Model (DSFM) for wall elements (models developed by Prof. Frank Vecchio and Michael Collins, University of Toronto).



$$[D_c] = \begin{bmatrix} \bar{E}_{c1} & 0 & 0 \\ 0 & \bar{E}_{c2} & 0 \\ 0 & 0 & \bar{G}_c \end{bmatrix} \quad [D_s]_i = \begin{bmatrix} \rho_i \bar{E}_{si} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\bar{E}_{c1} = \frac{f_{c1}}{\varepsilon_{c1}}; \bar{E}_{c2} = \frac{f_{c2}}{\varepsilon_{c2}}; \bar{G}_c = \frac{\bar{E}_{c1} \cdot \bar{E}_{c2}}{\bar{E}_{c1} + \bar{E}_{c2}} \quad \bar{E}_{si} = \frac{f_{si}}{\varepsilon_{si}}$$

$$[D_c] = [T_c]^T [D_c]' [T_c] + [T_f]^T [D_f]' [T_f]$$

$$[D_s]_i = [T_s]_i^T [D_s]_i' [T_s]_i$$

$$[T] = \begin{bmatrix} \cos^2 \psi & \sin^2 \psi & \cos \psi \cdot \sin \psi \\ \sin^2 \psi & \cos^2 \psi & -\cos \psi \cdot \sin \psi \\ -2 \cos \psi \cdot \sin \psi & 2 \cos \psi \cdot \sin \psi & (\cos^2 \psi - \sin^2 \psi) \end{bmatrix}$$

## REQUIREMENTS

The proposed solution consists in implementing the suite of calculation (Straus7) with an Add-on that executes nonlinear static analysis (Pushover analysis) for reinforced concrete buildings, with the following features:

- Possibility to perform nonlinear analysis of mixed wall-frame structures, in which it is possible to combine, through the sub-modeling technique, the nonlinearity of Straus7 with the secant stiffness formulation of the smeared crack models implemented for walls and slab elements.

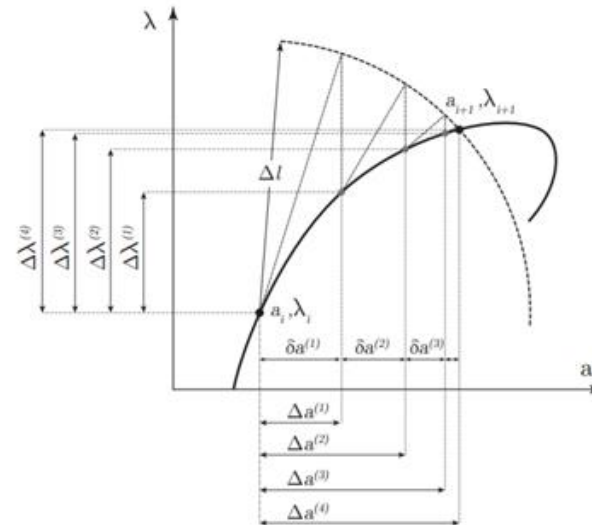


Figure 2.4: A schematic representation of the Arc-Length method iterations.  $a$  denotes a normalized displacement whereas  $\lambda$  the load incrementation parameter. The increment is defined by the radius of the circle  $\Delta l$  and the next point is the point of intersection between the path and the circle.

## REQUIREMENTS

The proposed solution consists in implementing the suite of calculation (Straus7) with an Add-on that executes nonlinear static analysis (Pushover analysis) for reinforced concrete buildings, with the following features:

- Possibility to perform multimodal adaptive pushover analysis, whose load distribution is continuously updated during the analysis, to reflect the progressive stiffness degradation of the structure.

- adaptive load distribution (belonging to Group 2), continuously updated during the analysis, to reflect the progressive stiffness degradation of the structure. This distribution considers the contribution of multiple modes of vibration, updated at each load step (*Adaptive Analysis* module):

$$F_i = \sqrt{\sum_n \sum_m \rho_{n,m} \cdot F_{i,n} \cdot F_{i,m}}$$

where

$$F_{ij} = \Gamma_j \cdot \phi_{i,j} \cdot M_i \cdot S_a(T_j, \xi)$$

$$\rho_{n,m} = \frac{8 \xi^2 \beta_{i,j}^{3/2}}{(1 + \beta_{i,j}) \cdot [(1 - \beta_{i,j})^2 + 4 \cdot \xi^2 \cdot \beta_{i,j}]}$$

being

$i$  = node index;  $j, n, m$  = index of vibration mode

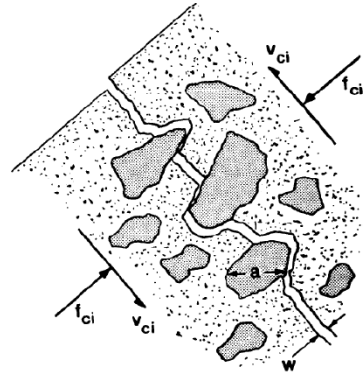
$\xi$  = damping factor for vibration modes

$\beta_{ij} = T_j/T_i$  = inverse ratio between the periods of each  $i$ - $j$  pair of modes

## NONLINEARITY OF THE STRUCTURE

### - Concrete constitutive law

The tensile bond  $\sigma$ - $\varepsilon$  of concrete is generally represented by a linear elastic branch up to the value of the tension equal to the maximum tensile strength  $f_t$ , and then continues with a softening branch, which represents the capacity of concrete, also unreinforced, to absorb pulling forces after cracking. This "residual" resistance is linked to the aggregate interlock. By increasing the intensity of the load, the crack opens and the contribution offered by this mechanism tends to disappear.



Shear across with crack through aggregate interlock

# NONLINEAR BEHAVIOR OF REINFORCED CONCRETE SHEAR WALLS



## NONLINEARITY OF THE STRUCTURE

- **Concrete constitutive law**

In the case of reinforced concrete, the softening branch can be modified by adding to the resistance offered by the aggregates the contribution due to the tensile stress absorbed by the concrete between two cracks for adherence with the steel bars.

This is the approach generally used in finite element analyses, which implement in the used algorithm a constitutive bond of the concrete in tension, able to model the phenomenon of tension stiffening.

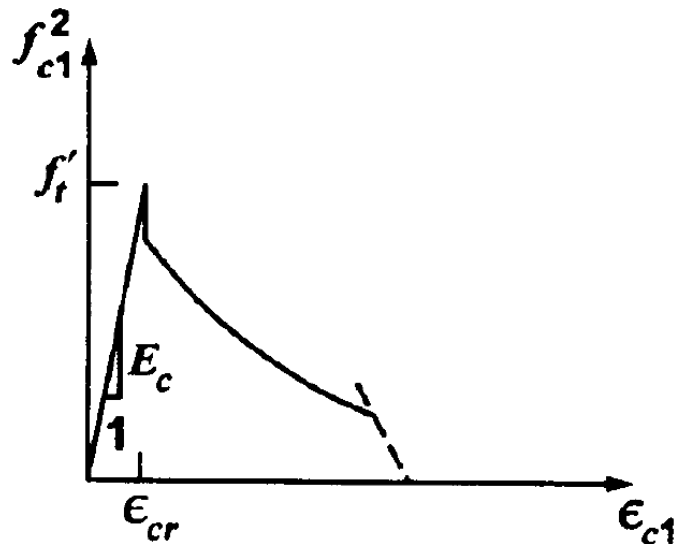


**NONLINEARITY OF THE STRUCTURE**

- **Concrete constitutive law**

*Model of tension stiffening proposed by (Vecchio, 2000)*

The constitutive bond of the concrete in tension consists of a first elastic branch and a softening branch whose analytical expression is:



$$f_{c1}^a = \frac{f_t'}{1 + \sqrt{c_t \epsilon_{c1}}} \quad \epsilon_{cr} < \epsilon_{c1}$$

$$c_t = 2.2m$$

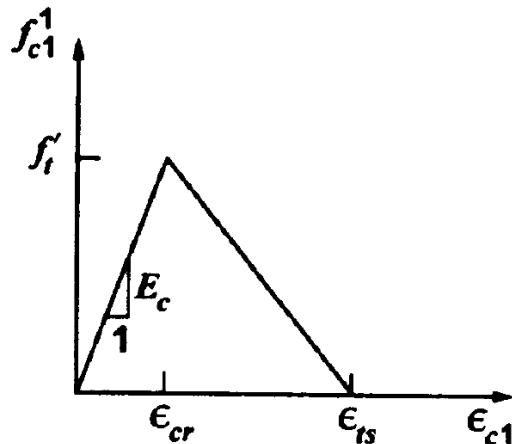
$$\frac{1}{m} = \sum_{i=1}^n \frac{4\rho_i}{d_{b_i}} |\cos \theta_{n_i}| \quad (\text{in mm})$$

## NONLINEARITY OF THE STRUCTURE

### - Concrete constitutive law

#### *Model of tension softening proposed by (Vecchio, 2000)*

This formulation is sufficient to describe the behavior of a concrete element or a weakly reinforced concrete element in which tension softening prevails over tension stiffening.



$$f_{c1} = \max(f_{c1}^a, f_{c1}^b)$$

After cracking, concrete can continue to carry tensile stresses as a result of two independent mechanisms: tension softening and tension stiffening. Tension softening refers to the fracture-associated mechanisms described by Darwin and others. It is particularly significant in concrete structures containing little or no reinforcement; for example, beams containing no web steel. Here, the concrete postcracking tensile stress associated with tension softening  $f_{c1}^a$  is calculated

$$f_{c1}^a = f_t' \left[ 1 - \frac{(\epsilon_{c1} - \epsilon_{cr})}{(\epsilon_{ts} - \epsilon_{cr})} \right] \quad (33)$$

where the terminal strain  $\epsilon_{ts}$  is calculated from the fracture energy parameter  $G_f$  and characteristic length  $L_r$ , as follows:

$$\epsilon_{ts} = 2.0 \frac{G_f}{f_t' \cdot L_r} \quad (34)$$

The parameter  $G_f$  is taken as having a constant value of 75 N/m. The resulting concrete tension softening formulation is illustrated in Fig. 9(b). This linear formulation is sufficient for most applications, but more accurate nonlinear models can be used.

NONLINEARITY OF THE STRUCTURE

- Concrete constitutive law

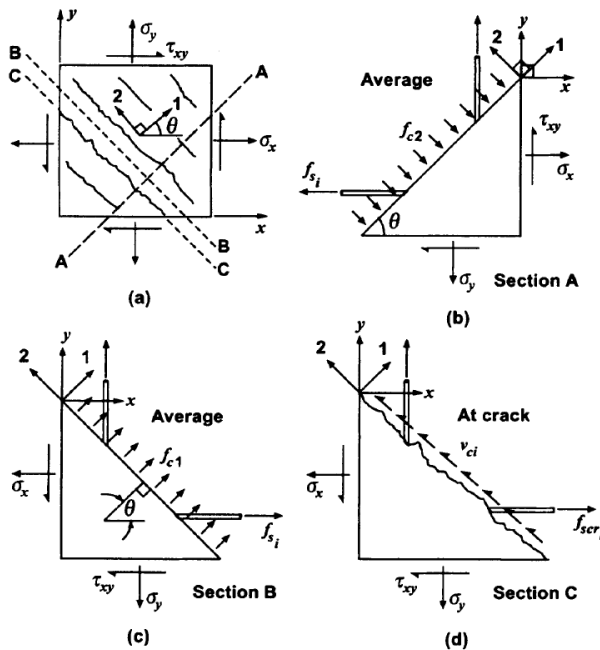


FIG. 6. Equilibrium Conditions: (a) External Conditions; (b) Perpendicular to Crack Direction; (c) Parallel to Crack Direction; (d) Along Crack Surface

Crack interfaces can be considered planes of weakness in the continuum, and it is necessary to check that the average stresses can be transmitted across the cracks. It will be assumed that the component of the concrete principal tensile stresses due to tension stiffening is zero at the crack location. To transmit the average stress  $f_{c1}$ , local increases in the reinforcement stresses are necessary. These local stresses are denoted as  $f_{scr_i}$ , as shown in Fig. 6(d). The magnitude of  $f_{c1}$  that can be transmitted via this mechanism is limited by the reserve capacity of the reinforcement, which is given by the difference between the average stresses and the yield stresses. Hence

$$f_{c1} \leq \sum_{i=1}^n \rho_i (f_{y_i} - f_{s_i}) \cdot \cos^2 \theta_{n_i} \quad (5)$$

where  $\rho_i$  = reinforcement ratio;  $f_{s_i}$  = average stress;  $f_{y_i}$  = yield stress for the  $i$ th reinforcement component; and the angle  $\theta_{n_i}$  = difference between the angle of orientation of the reinforcement,  $\alpha_i$ , and the normal to the crack surface  $\theta$ :

$$\theta_{n_i} = \theta - \alpha_i \quad (6)$$

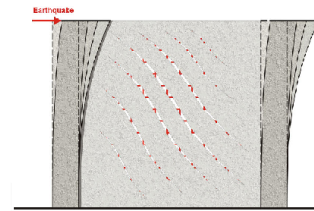
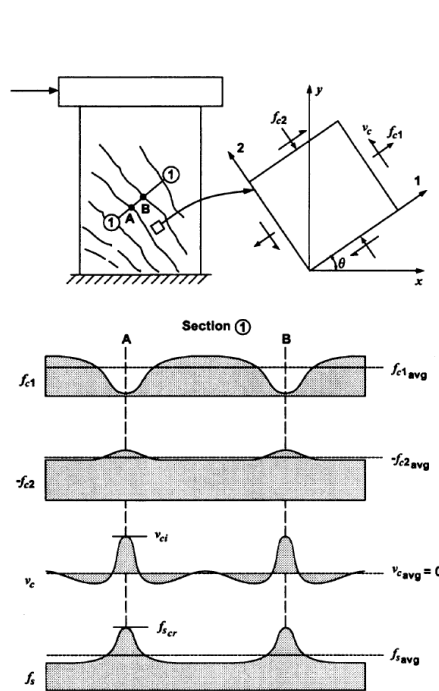
## NONLINEARITY OF THE STRUCTURE

### - Nonlinear behavior of reinforced concrete shear walls

The analytical model at the base of the module has been developed from the Modified Compression Field Theory (MCFT) and Disturbed Stress Field Model (DSFM), developed by Profs. Frank J. Vecchio, and Michael P. Collins (1986) of the University of Toronto, and is able to predict the response of reinforced concrete elements, representing the cracked concrete as an orthotropic material with rotating smeared cracks, while each steel Layer is represented as an orthotropic material able to resist only to uniaxial stresses in the bar direction of the same layer. The various layers of concrete and reinforcing bars are assembled to form an anisotropic elastic material.

**NONLINEARITY OF THE STRUCTURE**

- **Nonlinear behavior of reinforced concrete shear walls**



$$[D_c] = \begin{bmatrix} \bar{E}_{c1} & 0 & 0 \\ 0 & \bar{E}_{c2} & 0 \\ 0 & 0 & \bar{G}_c \end{bmatrix} \quad [D_s]_i' = \begin{bmatrix} \rho_i \bar{E}_{si} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\bar{E}_{c1} = \frac{f_{c1}}{\varepsilon_{c1}}; \bar{E}_{c2} = \frac{f_{c2}}{\varepsilon_{c2}}; \bar{G}_c = \frac{\bar{E}_{c1} \cdot \bar{E}_{c2}}{\bar{E}_{c1} + \bar{E}_{c2}} \quad \bar{E}_{si} = \frac{f_{si}}{\varepsilon_{si}}$$

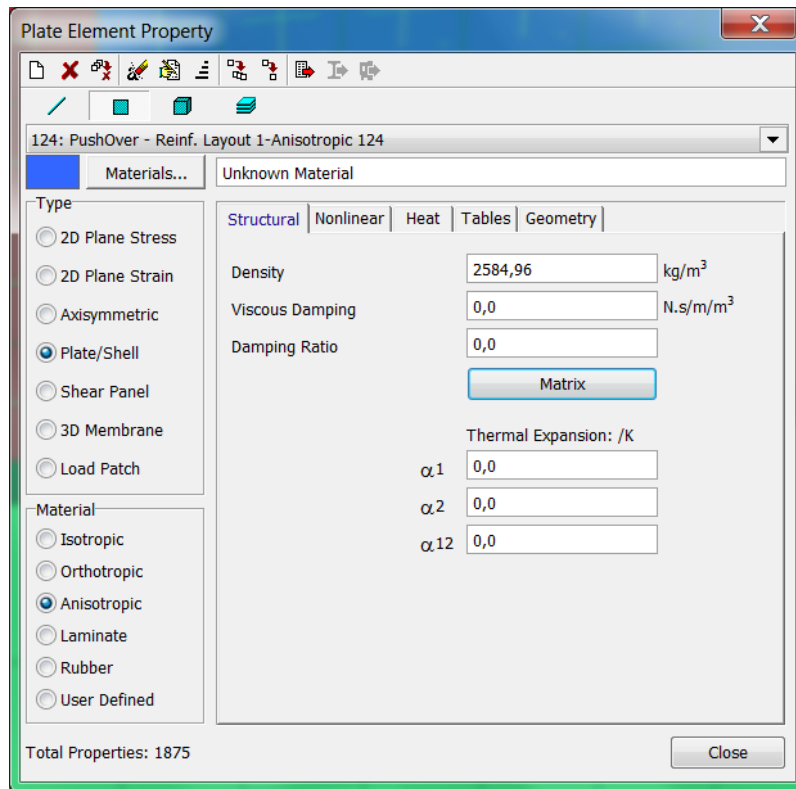
$$[D_c] = [T_c]^T [D_c]' [T_c] + [T_f]^T [D_f]' [T_f]$$

$$[D_s]_i = [T_s]_i^T [D_s]_i' [T_s]_i$$

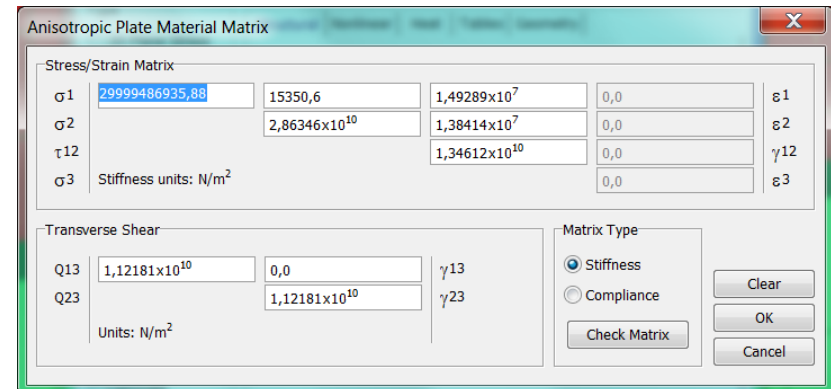
$$[T] = \begin{bmatrix} \cos^2 \psi & \sin^2 \psi & \cos \psi \cdot \sin \psi \\ \sin^2 \psi & \cos^2 \psi & -\cos \psi \cdot \sin \psi \\ -2 \cos \psi \cdot \sin \psi & 2 \cos \psi \cdot \sin \psi & (\cos^2 \psi - \sin^2 \psi) \end{bmatrix}$$

## NONLINEARITY OF THE STRUCTURE

- Nonlinear behavior of reinforced concrete shear walls



The elements used in Straus7 are *Anisotropic Plate/Shell* elements.



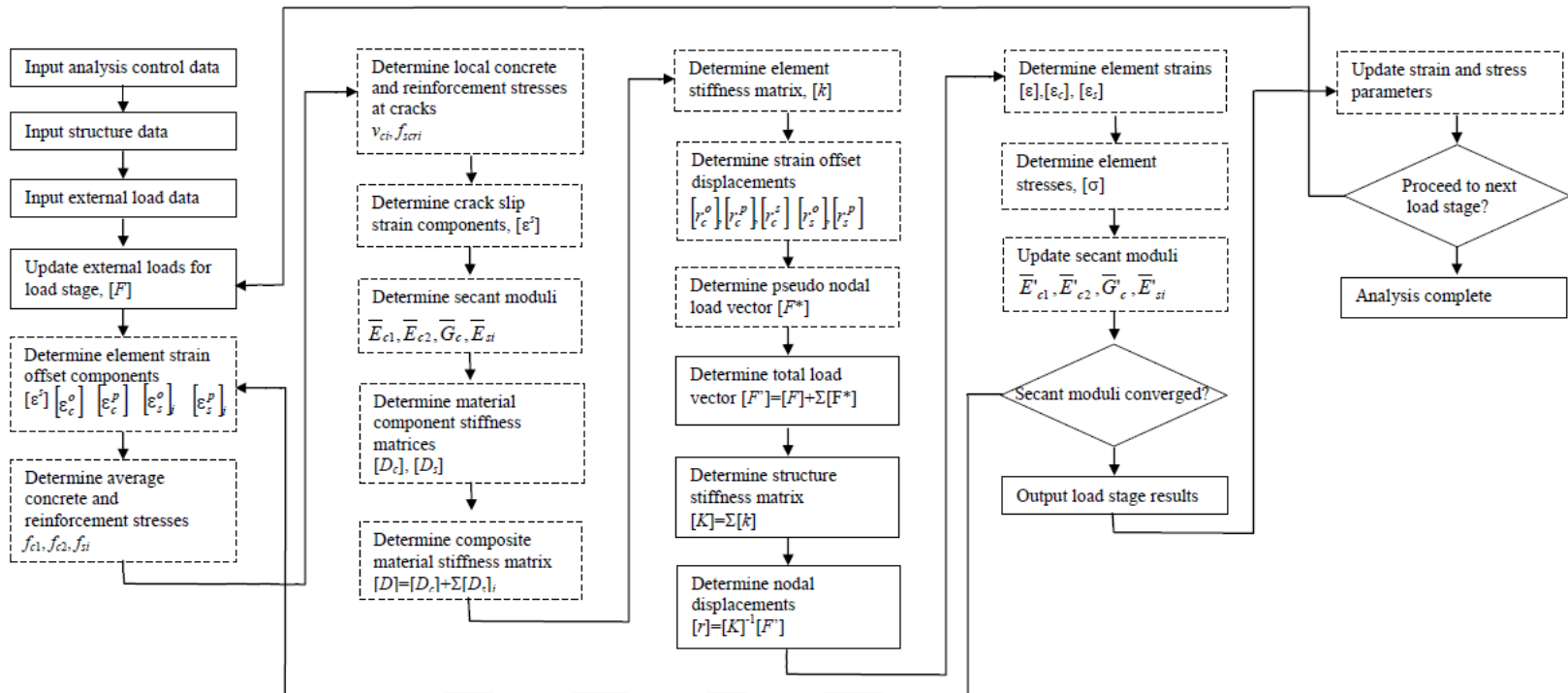
## NONLINEARITY OF THE STRUCTURE

### - **Nonlinear behavior of reinforced concrete shear walls**

The nonlinear calculation is obtained by adopting an iterative secant stiffness algorithm, in which the stiffness of the elastic anisotropic materials defined by EasyOver are updated at each iteration into a submodel created for the purpose (in which the displacements obtained in the global model are imposed to the plate elements). Once the convergence in the sub model is achieved, EasyOver updates the main model, by assigning the stiffness matrices obtained by the iterative calculation on the sub-model to the plate elements, applies a new load increment for the current stage, such as to obtain the same displacements obtained in the previous application, then it passes to the next load increment.

NONLINEARITY OF THE STRUCTURE

- Nonlinear behavior of reinforced concrete shear walls



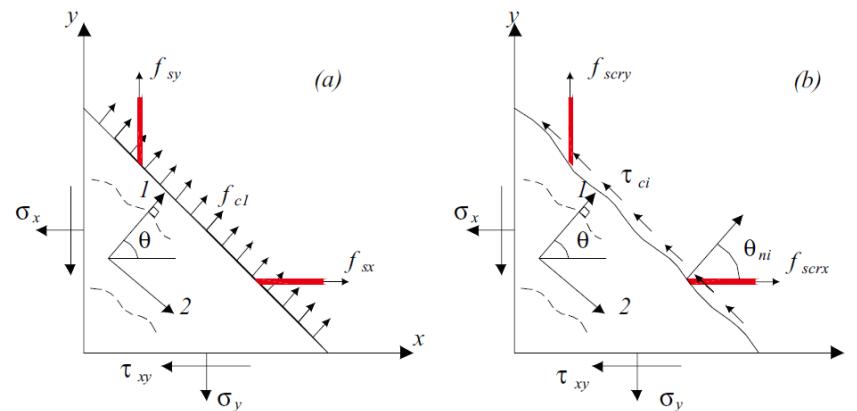


## NONLINEARITY OF THE STRUCTURE

### - Nonlinear behavior of reinforced concrete shear walls

The modeling of the cracked concrete element is performed by using an orthotropic material and a rotating smeared crack model. The cracked concrete is treated like a continuous medium with cracks smeared across the membrane element.

This formulation evaluates the average stresses and strains (in the region between the cracks) and the local stresses and strains of the concrete and reinforcement, as well as the widths and orientations of the cracks during the loading. Based on this information, the failure mode of the element can be determined.

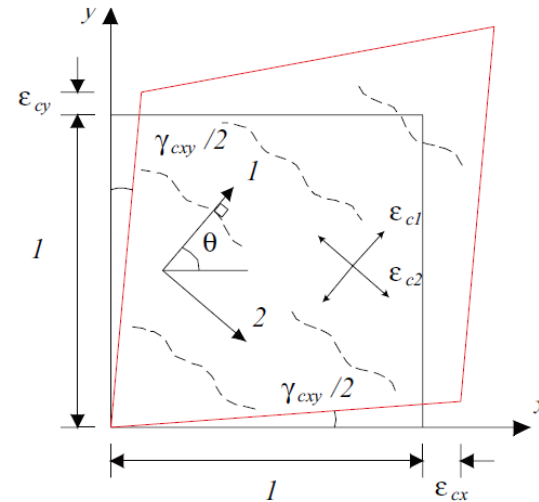
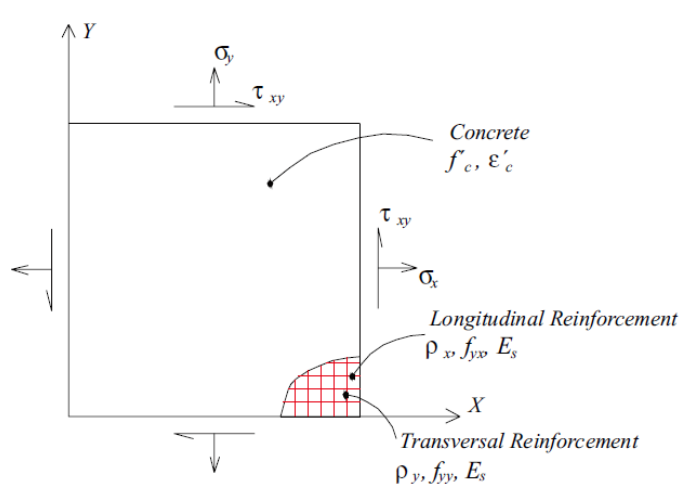


## NONLINEARITY OF THE STRUCTURE

### - Nonlinear behavior of reinforced concrete shear walls

The model is based on four groups of relationships:

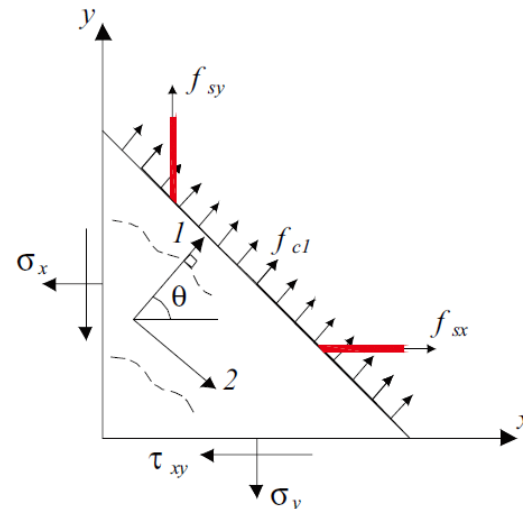
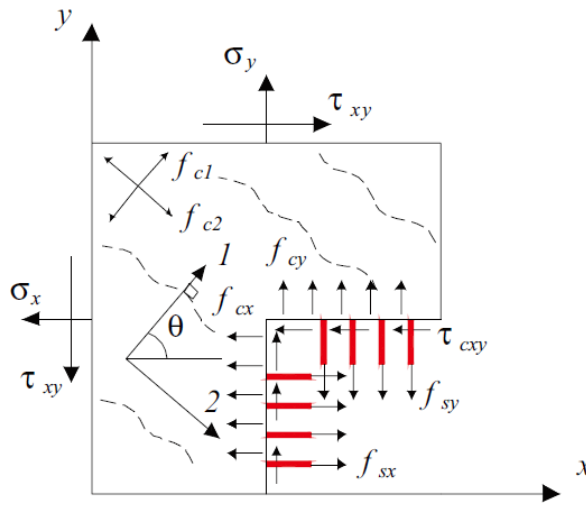
- compatibility relationships for the average strains in the concrete and reinforcement



**NONLINEARITY OF THE STRUCTURE**

- **Nonlinear behavior of reinforced concrete shear walls**

- equilibrium relationships for the average stresses in the concrete and reinforcement

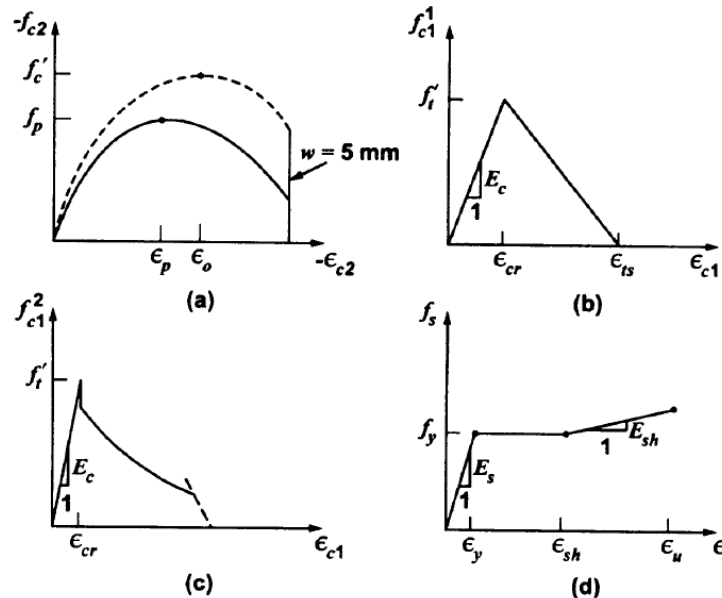


# NONLINEAR BEHAVIOR OF REINFORCED CONCRETE SHEAR WALLS



## NONLINEARITY OF THE STRUCTURE

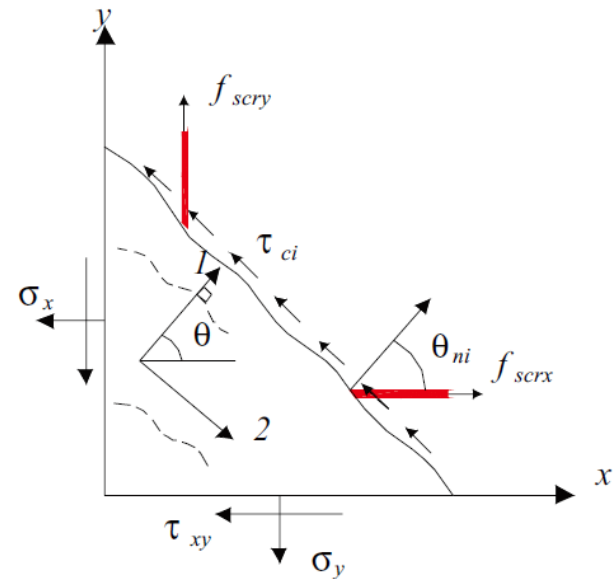
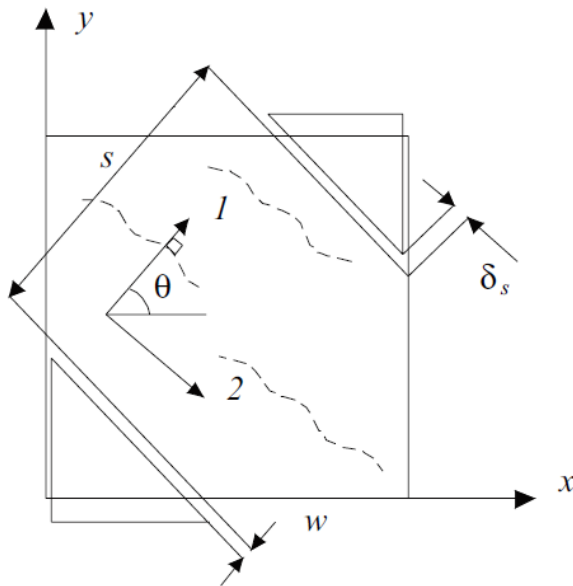
- Nonlinear behavior of reinforced concrete shear walls
- constitutive relationships for the cracked concrete and reinforcement



Constitutive Relations: (a) Compressive Softening Model; (b) Tension Softening Model; (c) Tension Stiffening Model; (d) Reinforcing Steel Response

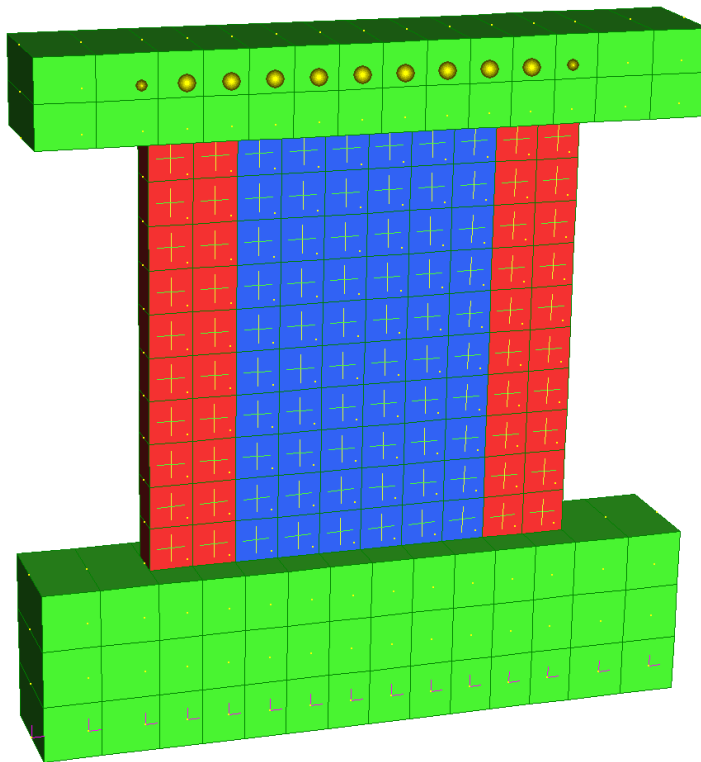
**NONLINEARITY OF THE STRUCTURE**

- **Nonlinear behavior of reinforced concrete shear walls**
- compatibility relationships for crack shear slip



## NONLINEARITY OF THE STRUCTURE

### - Nonlinear behavior of reinforced concrete shear walls



Model with a wall of dimensions 75x75 cm and 7 cm thick, at the top of which non-structural masses are applied for a total of 46,000 kg. The wall will be loaded by lateral forces lying on its plane, according to the X axis of the global model.

Area 1 (central plates):

- d.6/8 cm in dir. of local axis x (horiz. reinf.)
- d.8/6 cm in dir. of local axis y (vert. reinf.)

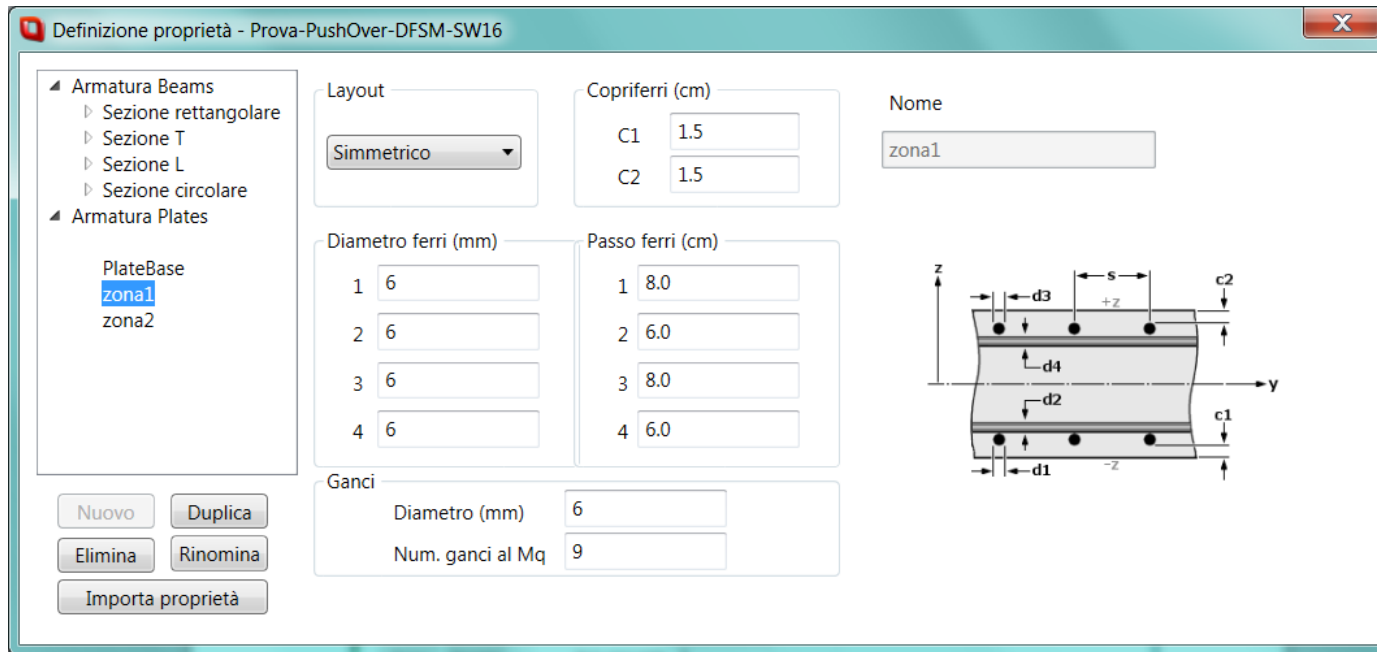
Area 2 (wall end plates):

- d.6/8 cm in dir. of local axis x (horiz. reinf.)
- d.8/5 cm in dir. of local axis y (vert. reinf.)
- d.4/5 cm in dir. of local axis y (stirrups)

Concrete cover = 15 mm

## NONLINEARITY OF THE STRUCTURE

### - Nonlinear behavior of reinforced concrete shear walls



In the sub menu *Plate reinforcement* of EasyOver you can create and define properties for the reinforcement of plate elements. The screen is quite similar to the one in Straus7 for Plate RC properties.

## NONLINEARITY OF THE STRUCTURE

### - Nonlinear behavior of reinforced concrete shear walls

Out-of-plane stresses and deformation, due to the confinement of lateral expansion by out-of-plane reinforcement (No. hooks / sq)

$$\varepsilon_{cz} = \frac{E_c}{E_c + \rho_z \cdot E_{sz}} \left( -\nu_{12} \frac{f_{c2}}{E_{c2}} - \nu_{21} \frac{f_{c1}}{E_{c1}} \right)$$

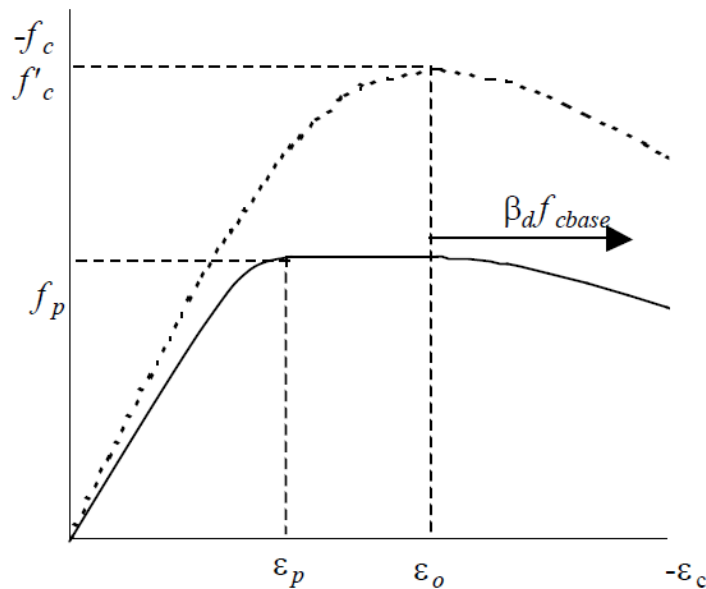
$$\varepsilon_{cz} = -\frac{\rho_z \cdot f_{z,yield}}{E_c} - \nu_{12} \frac{f_{c2}}{E_{c2}} - \nu_{21} \frac{f_{c1}}{E_{c1}}$$

$$f_{sz} = E_s \varepsilon_{cz} \leq f_{z,yield} \quad f_{cz} = -\rho_z \cdot f_{sz}$$



## NONLINEARITY OF THE STRUCTURE

- Nonlinear behavior of reinforced concrete shear walls



$$f_p = \beta_d \beta_l f'_c$$

$$\varepsilon_p = \beta_d \beta_l \varepsilon_o$$

$\beta_d \rightarrow$  compression softening  
 $\beta_l \rightarrow$  confinement

## NONLINEARITY OF THE STRUCTURE

### - Nonlinear behavior of reinforced concrete shear walls

ACI STRUCTURAL JOURNAL

TECHNICAL PAPER

Title no. 87-S3

**Behavior of Reinforced Concrete Structural Walls: Strength, Deformation Characteristics, and Failure Mechanism**



Ioannis D. Lefas, Michael D. Kotsovos, and Nicholas N. Ambraseys

**Table 1 — Properties of reinforcement bars**

Type	Yield strength $f_{sy}$ , MPa	Ultimate strength $f_{su}$ , MPa
8 mm high-tensile bar	470	565
6.25 mm high-tensile bar	520	610
4 mm mild-steel bar	420	490

1 mm = 0.0394 in.; 1 MPa = 145 psi.

Model with a wall of dimensions 75x75 cm and 7 cm thick, at the top of which non-structural masses are applied for a total of 46,000 kg. The wall will be loaded by lateral forces lying on its plane, according to the X axis of the global model.

Area 1 (central plates):

- d.6/8 cm in dir. of local axis x (horiz. reinf.)
- d.8/6 cm in dir. of local axis y (vert. reinf.)

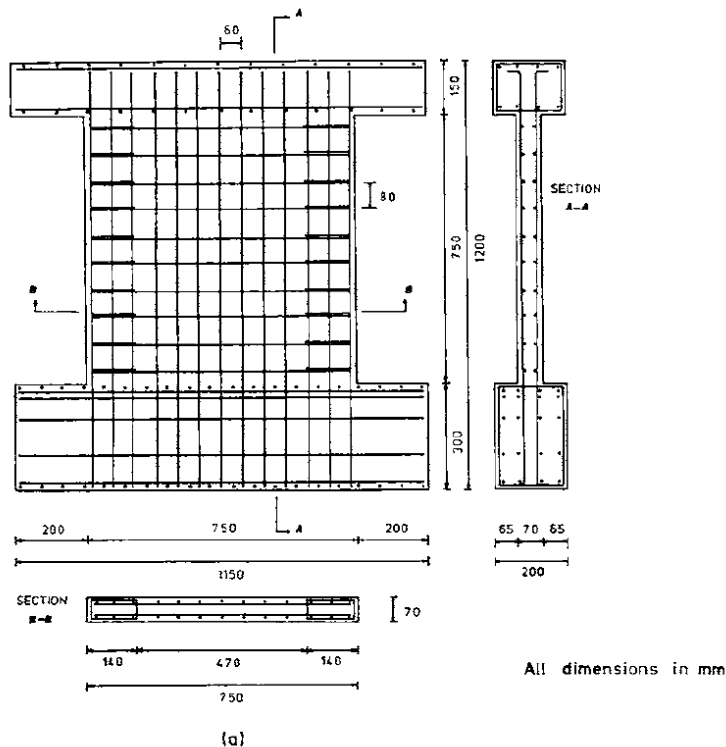
Area 2 (wall end plates):

- d.6/8 cm in dir. of local axis x (horiz. reinf.)
- d.8/5 cm in dir. of local axis y (vert. reinf.)
- d.4/5 cm in dir. of local axis y (stirrups)

Concrete cover = 15 mm

## NONLINEARITY OF THE STRUCTURE

### - Nonlinear behavior of reinforced concrete shear walls



Model with a wall of dimensions 75x75 cm and 7 cm thick, at the top of which non-structural masses are applied for a total of 46,000 kg. The wall will be loaded by lateral forces lying on its plane, according to the X axis of the global model.

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Area 2 (wall end plates):

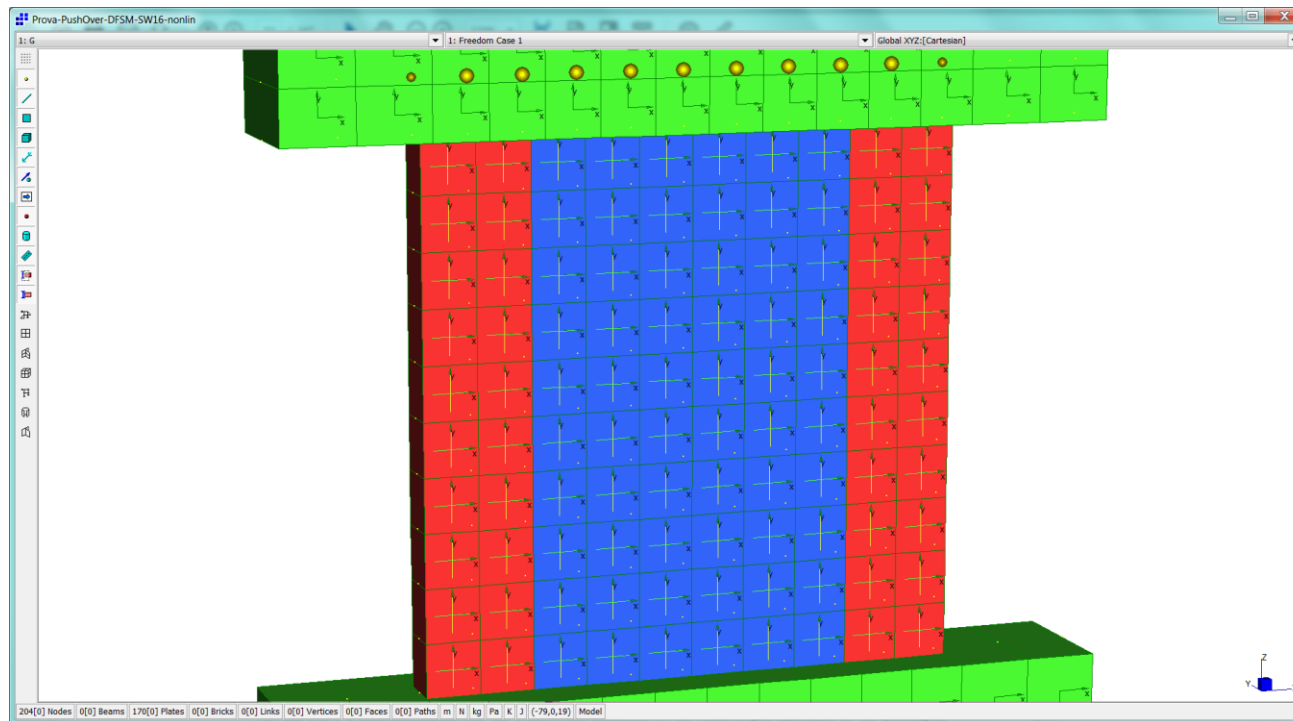
- d.6/8 cm in dir. of local axis x (horiz. reinf.)
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## NONLINEARITY OF THE STRUCTURE

- **Nonlinear behavior of reinforced concrete shear walls**

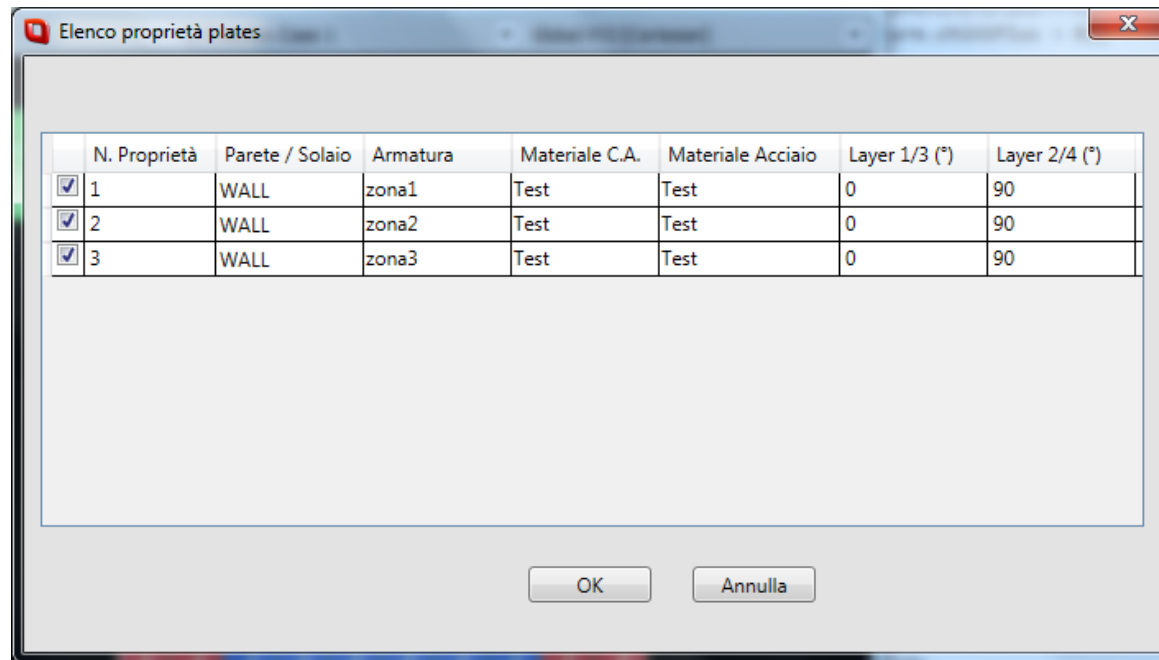
It is important to correctly orient the various plate elements in the Straus7 model.



## NONLINEARITY OF THE STRUCTURE

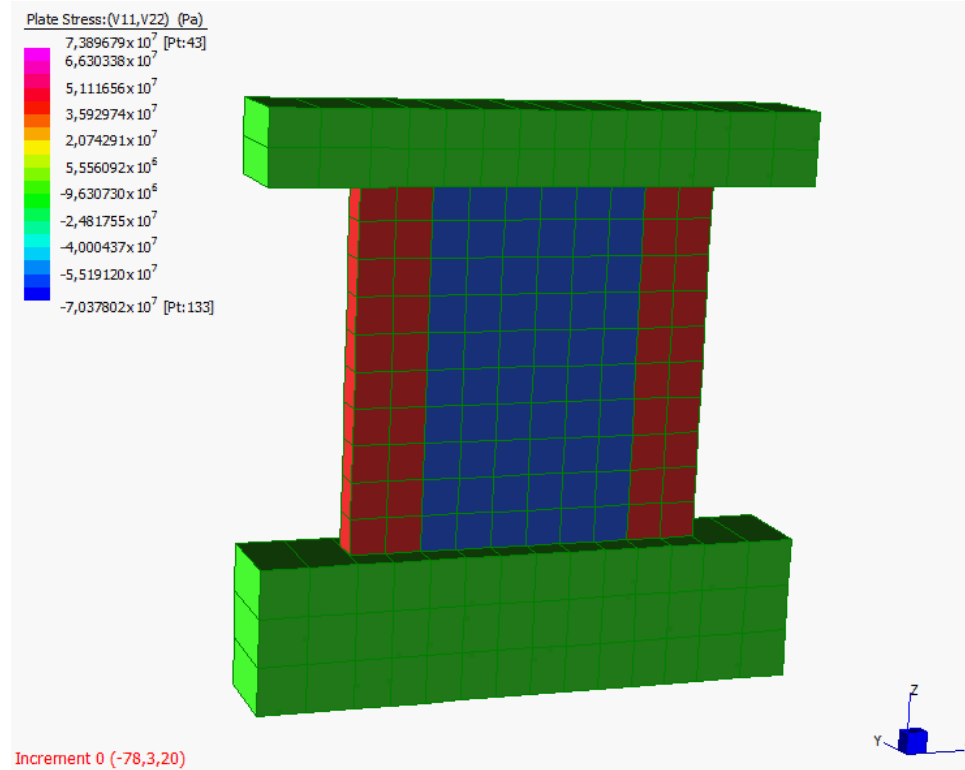
### - Nonlinear behavior of reinforced concrete shear walls

You can assign to the various Straus7 plate properties the properties for the reinforcement defined in the menu just seen, as well as the concrete and steel material.



## NONLINEARITY OF THE STRUCTURE

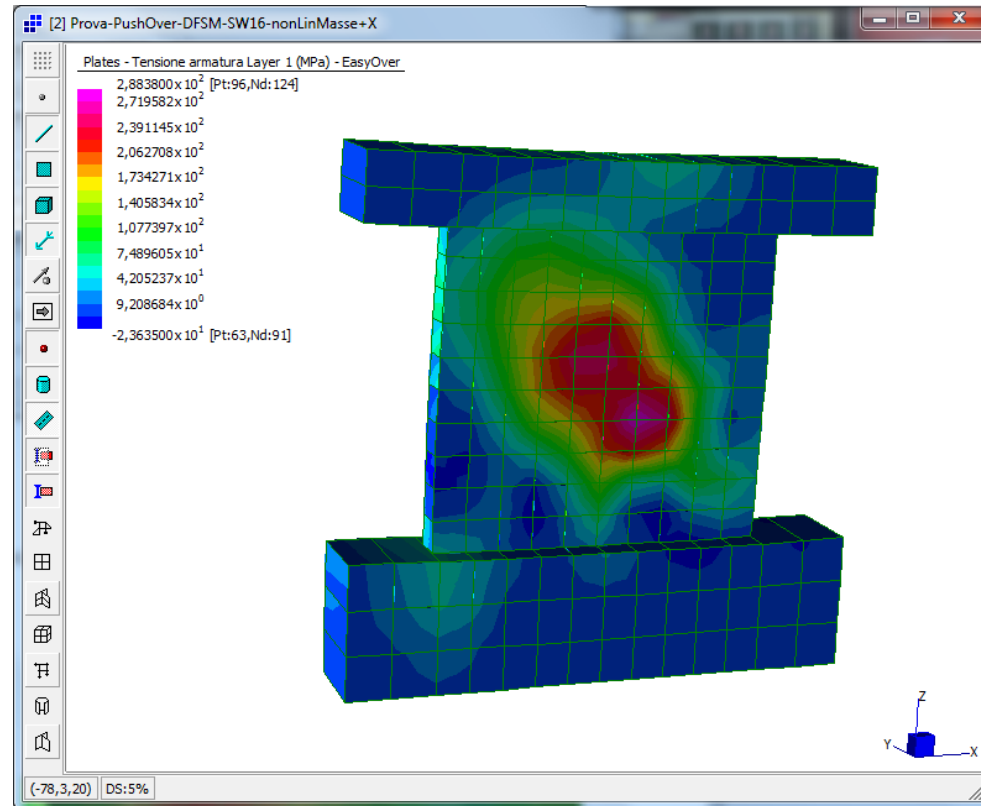
- Nonlinear behavior of reinforced concrete shear walls



## NONLINEARITY OF THE STRUCTURE

- Nonlinear behavior of reinforced concrete shear walls

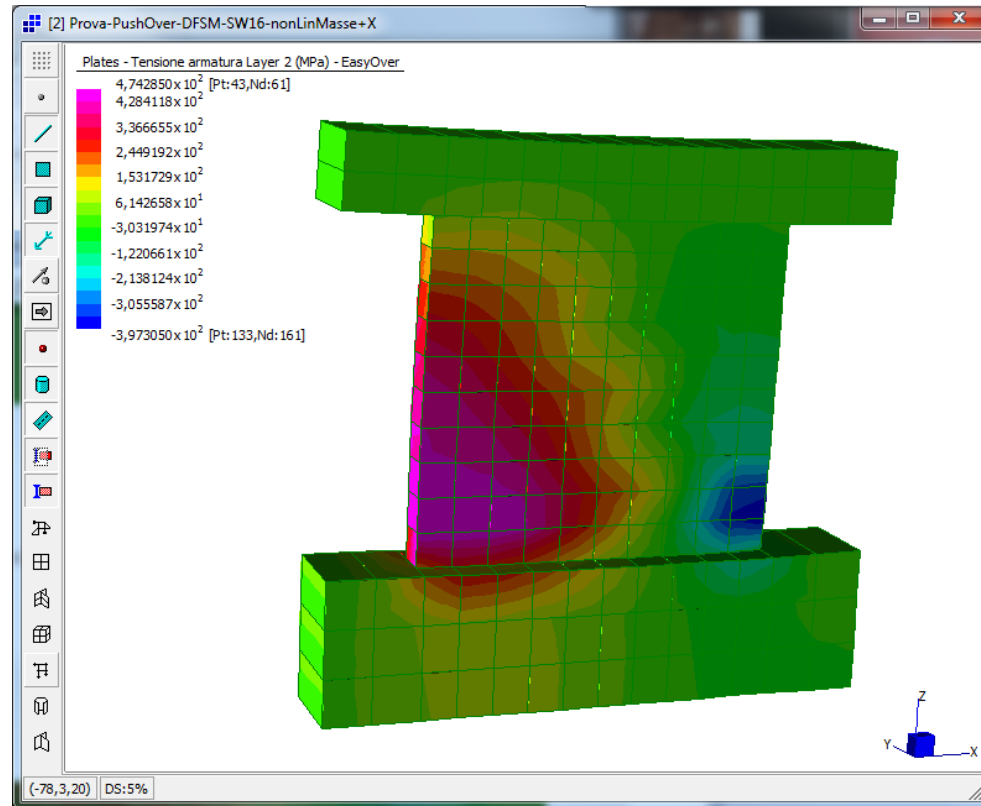
*Stress in the transverse reinforcement (layers 1 and 3).  
Maximum stress of 288 MPa (tensile stress).*



## NONLINEARITY OF THE STRUCTURE

- Nonlinear behavior of reinforced concrete shear walls

*Stress in the vertical reinforcement (layers 2 and 4).  
Maximum stress of 474 MPa (tensile stress).*

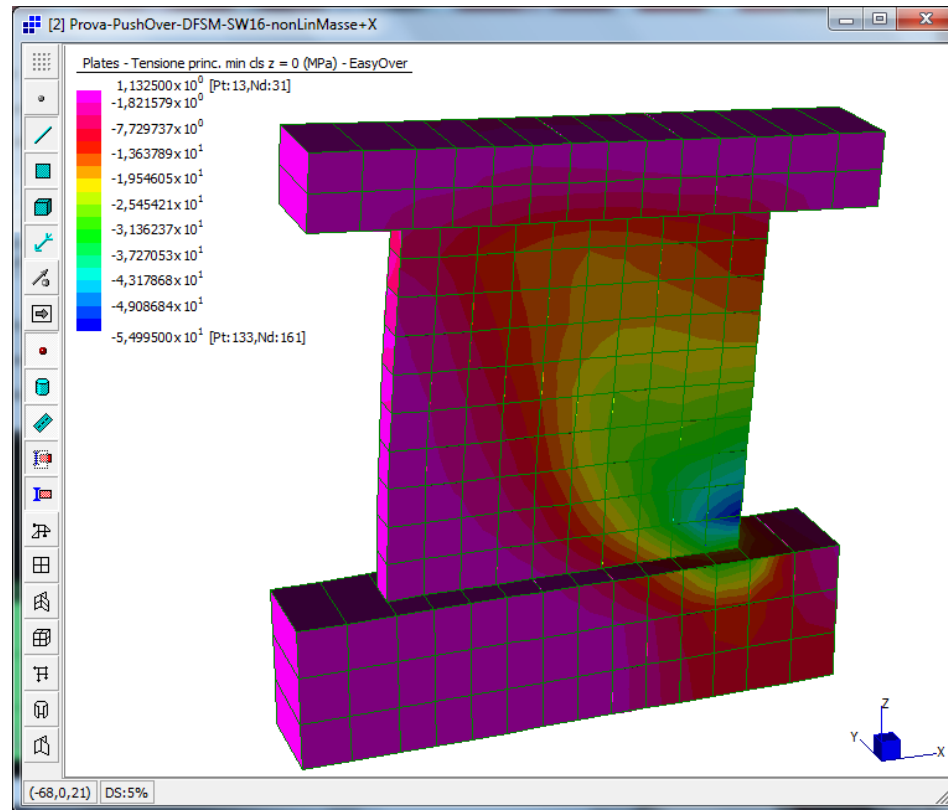




## NONLINEARITY OF THE STRUCTURE

- Nonlinear behavior of reinforced concrete shear walls

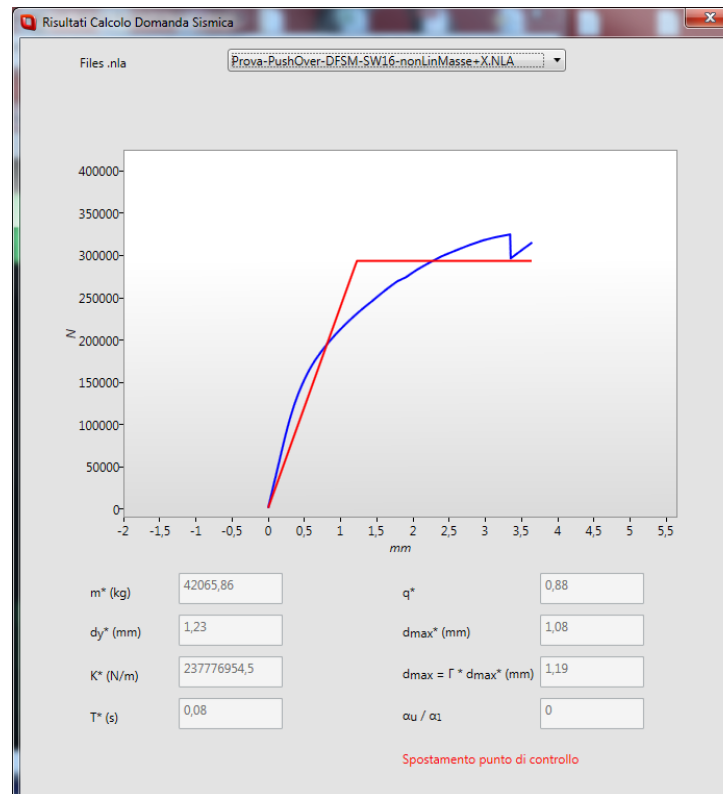
*Compressive stress in  
the concrete.  
Maximum stress of  
55 MPa.*



## NONLINEARITY OF THE STRUCTURE

- Nonlinear behavior of reinforced concrete shear walls

**Maximum base shear equal to 354.903 N**  
**Failure by web crushing, which occurs simultaneously with the yield of the reinforcement.**



## NONLINEARITY OF THE STRUCTURE

- Nonlinear behavior of reinforced concrete shear walls

*Comparison with the experimental results obtained from the test campaign carried out from Lefas et al. (1990): good correspondence*

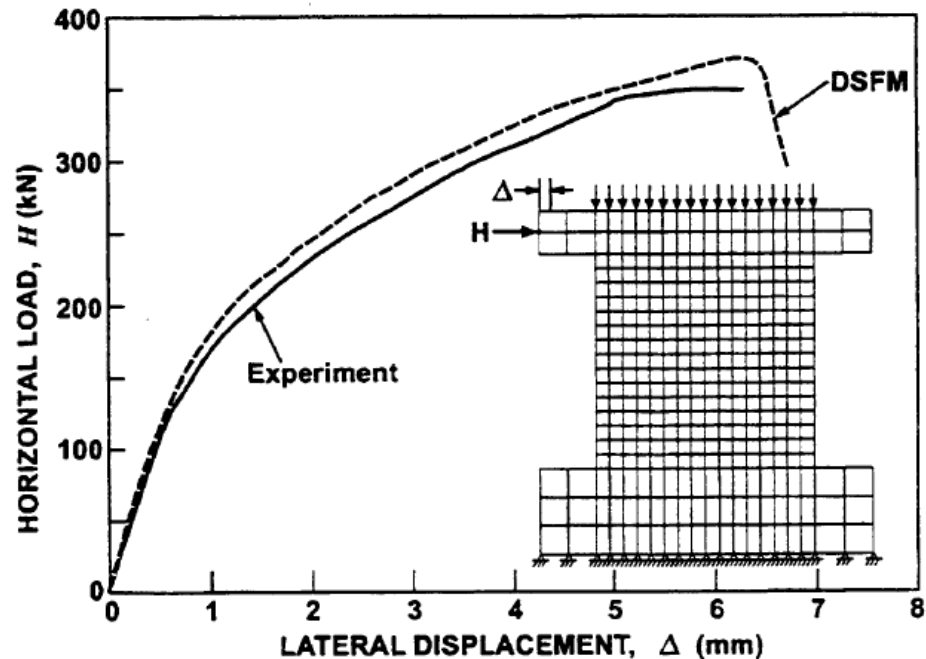


FIG. 11. Results of Finite-Element Analysis of Wall SW16

## NONLINEARITY OF THE STRUCTURE

### - Nonlinear behavior of reinforced concrete shear walls

*Comparison  
with analytical  
models for  
calculation of  
shear strength.*

STRUT WITH $\cot\theta = 1$		BISKINIS WITH $\mu_{\Delta}^{PL} = 5$		BISKINIS WITH $\mu_{\Delta}^{PL} = 0$	
b (mm)	70	b (mm)	70	b (mm)	70
h (mm)	750	h (mm)	750	h (mm)	750
c (mm)	15	c (mm)	15	c (mm)	15
$f_c$ (N/mm <sup>2</sup> )	49	x (mm)	525,00	x (mm)	525,00
$f_y$ (N/mm <sup>2</sup> )	470	$L_v$ (mm)	825	$L_v$ (mm)	825
$A_{stirrups}$ (mm <sup>2</sup> )	56,52	$f_c$ (N/mm <sup>2</sup> )	49	$f_c$ (N/mm <sup>2</sup> )	49
s (mm)	67	$f_y$ (N/mm <sup>2</sup> )	470	$f_y$ (N/mm <sup>2</sup> )	470
v	0,50	$\mu_{\Delta}^{PL}$	5,00	$\mu_{\Delta}^{PL}$	-
N (N)	460.000	$A_{long}$ (mm <sup>2</sup> )	301,44	$A_{long}$ (mm <sup>2</sup> )	301,44
$\omega_{sw}$	0,11531	$A_{stirrups}$ (mm <sup>2</sup> )	56,52	$A_{stirrups}$ (mm <sup>2</sup> )	56,52
$\sigma_{cp}$ (N/mm <sup>2</sup> )	8,76	s (mm)	67	s (mm)	67
$\alpha_c$	1,18	N (N)	460.000	N (N)	460.000
$\cot\theta^*$	1,00	$P_{tot}$	0,00574	$P_{tot}$	0,00574
$\cot\theta$	1,00	$\rho_w$	0,01205	$\rho_w$	0,01205
$V_{Rsd}$ (N)	262.274	$A_c$ (mm <sup>2</sup> )	51.450	$A_c$ (mm <sup>2</sup> )	51.450
$V_{Rcd}$ (N)	670.055	$V_N$ (N)	62.727	$V_N$ (N)	62.727
$V_R$ (N)	<b>262.274</b>	k	0,75	k	1,00
		$V_c$ (N)	27.296	$V_c$ (N)	27.296
		$V_w$ (N)	262.274	$V_w$ (N)	262.274
		$V_{R,max}$ (N)	274.967	$V_{R,max}$ (N)	392.810
		$V_R$ (N)	<b>274.967</b>	$V_R$ (N)	<b>352.297</b>

## NONLINEARITY OF THE STRUCTURE

### - Nonlinear behavior of reinforced concrete shear walls

(2) The shear strength of a concrete wall,  $V_R$ , may not be taken greater than the value corresponding to failure by web crushing,  $V_{R,max}$ , which under cyclic loading may be calculated from the following expression (with units: MN and meters):

$$V_{R,max} = \frac{0,85(1 - 0,06 \min(5; \mu_{\Delta}^{pl}))}{\gamma_{el}} \left( 1 + 1,8 \min(0,15; \frac{N}{A_c f_c}) \right) \left( 1 + 0,25 \max(1,75; 100 \rho_{tor}) \right) \left( 1 - 0,2 \min(2; \frac{L_V}{h}) \right) \sqrt{f_c} b_w z \quad (A.15)$$

where  $\gamma_{el} = 1,15$  for primary seismic elements and 1,0 for secondary seismic ones,  $f_c$  is in MPa,  $b_w$  and  $z$  are in meters and  $V_{R,max}$  in MN, and all other variables are as defined in (1).

The shear strength under cyclic loading as controlled by web crushing prior to flexural yielding is obtained from expression (A.15) for  $\mu_{\Delta}^{pl}=0$ .

**Comparison  
with analytical  
models for  
calculation of  
shear strength.  
EC8 - Part 3**

## NONLINEARITY OF THE STRUCTURE

### - Nonlinear behavior of reinforced concrete shear walls

*Comparison  
with analytical  
models for  
calculation of  
shear strength.*

VARIABLE STRUT INCLINATION METHOD		CIRC. 617/2009	
b (mm)	70	b (mm)	70
h (mm)	750	h (mm)	750
c (mm)	15	c (mm)	15
$f_c$ (N/mm <sup>2</sup> )	49	$f_c$ (N/mm <sup>2</sup> )	49
$f_y$ (N/mm <sup>2</sup> )	470	$f_y$ (N/mm <sup>2</sup> )	470
$A_{stirrups}$ (mm <sup>2</sup> )	56,52	$A_{long}$ (mm <sup>2</sup> )	301,44
s (mm)	67	$A_{stirrups}$ (mm <sup>2</sup> )	56,52
$\nu$	0,50	s (mm)	67
N (N)	460.000	N (N)	460.000
$\omega_{sw}$	0,11531	$\rho_l$	0,00586
$\sigma_{cp}$ (N/mm <sup>2</sup> )	8,76	$\sigma_{cp}$ (N/mm <sup>2</sup> )	8,76
$\alpha_c$	1,18	$\alpha_c$	1,18
$\cot\theta^*$	2,03	k	1,52
$\cot\theta$	2,03	$v_{min}$ (N/mm <sup>2</sup> )	0,460
$V_{Rsd}$ (N)	531.685	$V_{Rc}$ (N)	96.410
$V_{Rcd}$ (N)	531.685	$V_{Rsd}$ (N)	262.274
$V_R$ (N)	<b>531.685</b>	$V_{Rd,max}$ (N)	670.055
		$V_R$ (N)	<b>358.684</b>

NONLINEAR ANALYSIS OF MIXED WALL-FRAME STRUCTURES

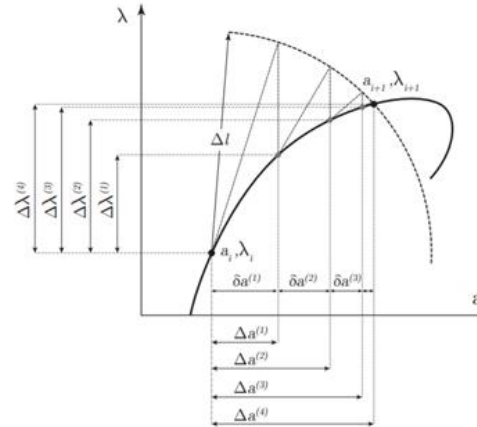
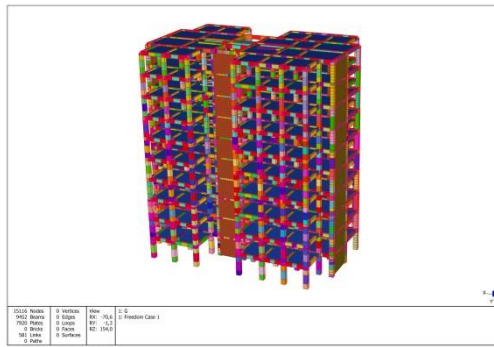
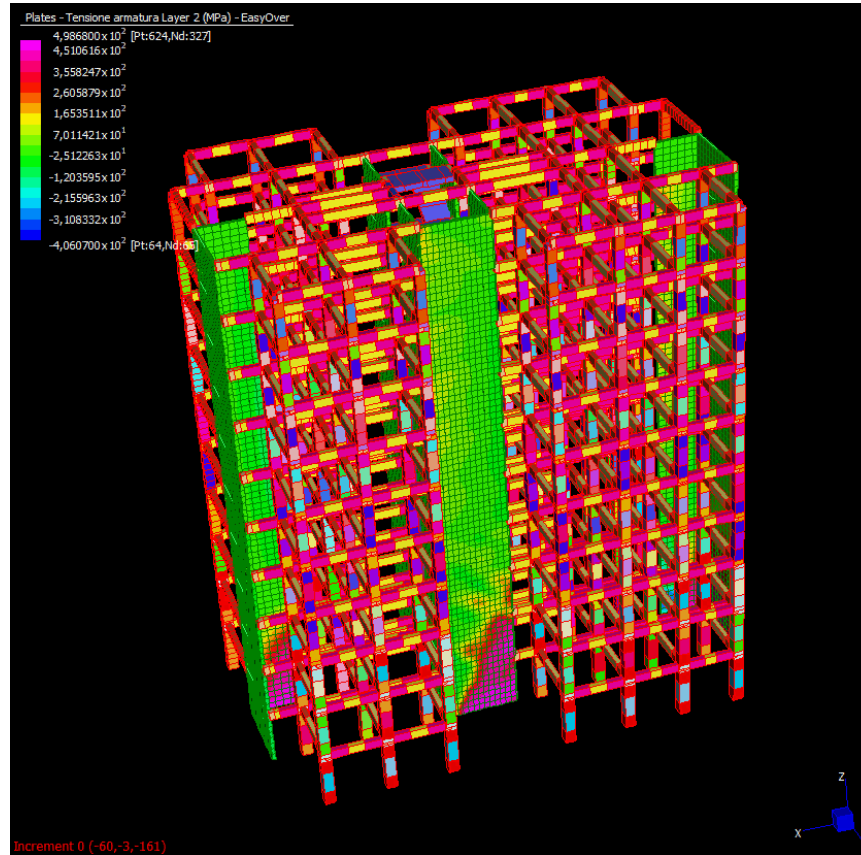


Figure 2.4: A schematic representation of the Arc-Length method iterations.  $a$  denotes a normalized displacement whereas  $\lambda$  the load increment parameter. The increment is defined by the radius of the circle  $\Delta l$  and the next point is the point of intersection between the path and the circle.



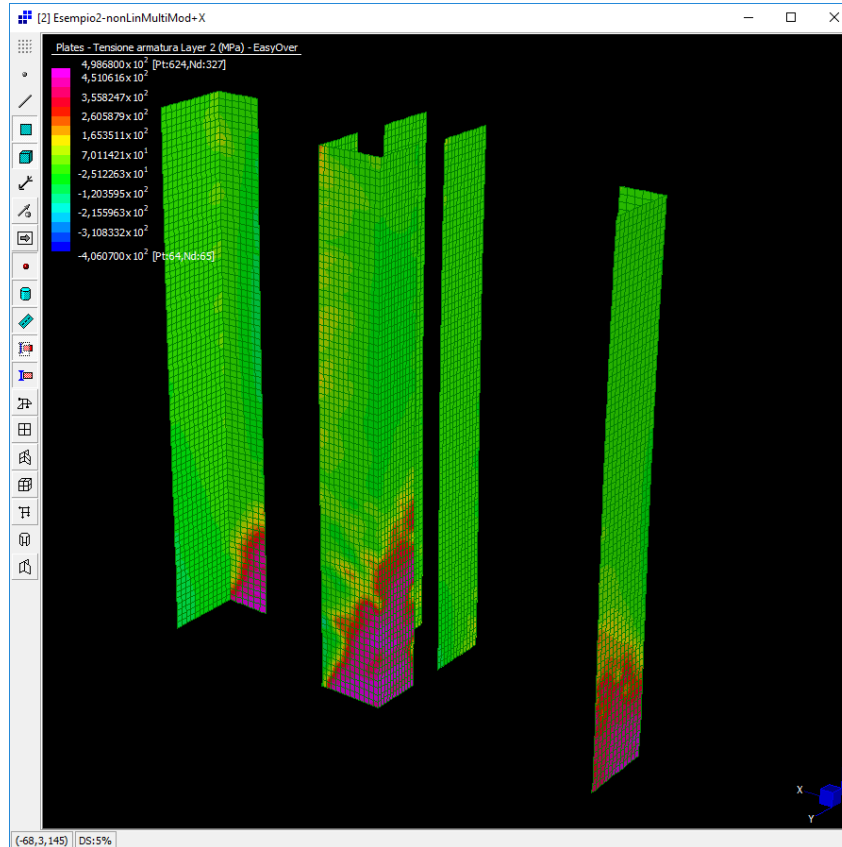
**NONLINEAR ANALYSIS OF MIXED WALL-FRAME STRUCTURES**





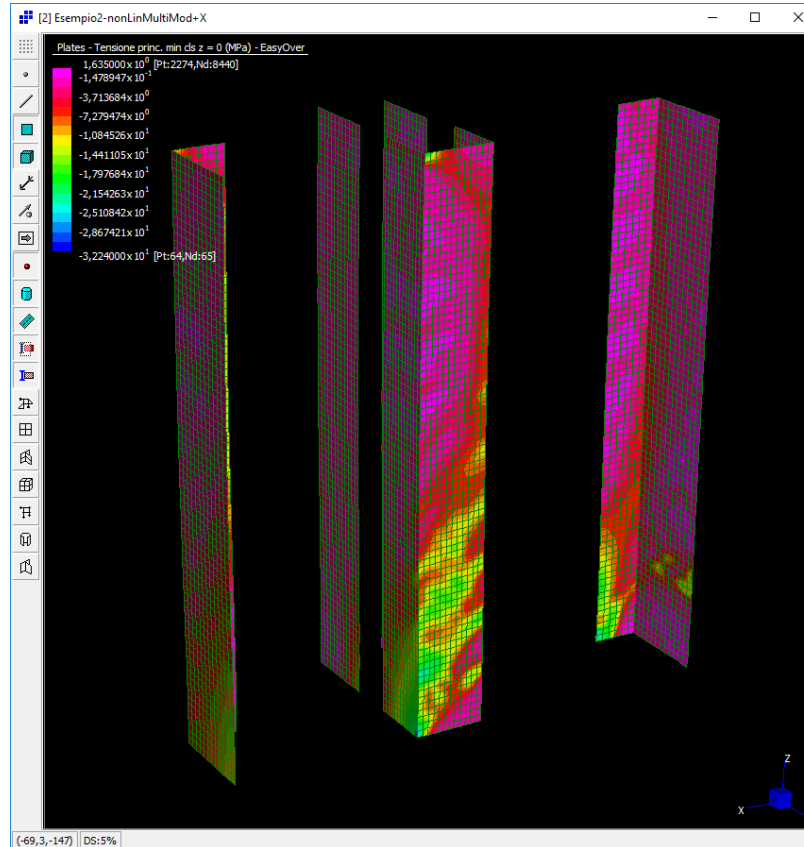
**NONLINEAR ANALYSIS OF MIXED WALL-FRAME STRUCTURES**

*Stress in the vertical  
reinforcement (layers  
2 and 4).*

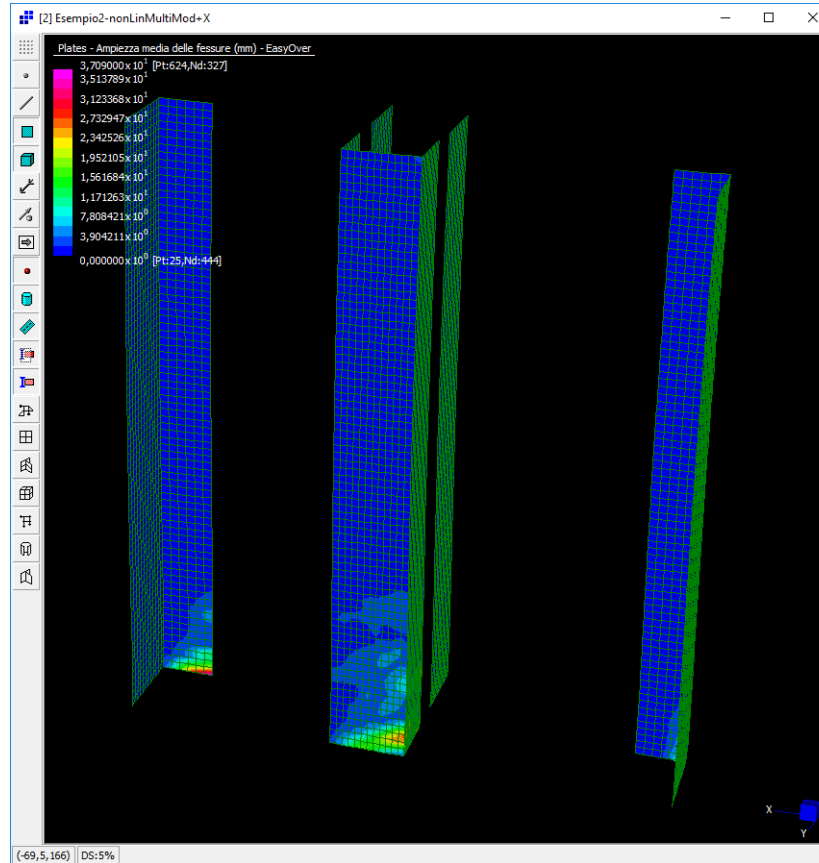


**NONLINEAR ANALYSIS OF MIXED WALL-FRAME STRUCTURES**

*Compressive stress in  
the concrete.*



**NONLINEAR ANALYSIS OF MIXED WALL-FRAME STRUCTURES**



*Average width of the  
cracks.*

## LATERAL LOAD DISTRIBUTIONS ADOPTED IN EASYOVER

- distribution proportional to the fundamental mode in the direction considered (belonging to Group 1):

$$F_i = \frac{m_i \cdot \phi_{fond,i}}{\sum_{j=1}^N m_j \cdot \phi_{fond,j}} V_b$$

- distribution calculated by combining modal responses from a response spectrum analysis of the building (belonging to Group 1):

$$F_i = \sqrt{\sum_n \sum_m \rho_{n,m} \cdot F_{i,n} \cdot F_{i,m}}$$

where

$$F_{ij} = \Gamma_j \cdot \phi_{i,j} \cdot M_i \cdot S_a(T_j, \xi)$$

$$\rho_{n,m} = \frac{8\xi^2 \beta_{i,j}^{3/2}}{(1 + \beta_{i,j}) \cdot [(1 - \beta_{i,j})^2 + 4 \cdot \xi^2 \cdot \beta_{i,j}]}$$

## LATERAL LOAD DISTRIBUTIONS ADOPTED IN EASYOVER

- analysis of different modes of vibration in the case of irregular buildings and calculation of the load profile corresponding to the equivalent modal shape (Valles, 1996) (belonging to Group 2):

$$F_i = \frac{m_i \cdot \phi_{eq,i}}{\sum_{j=1}^N m_j \cdot \phi_{eq,j}} V_b \quad \text{dove} \quad \phi_{eq,i} = \sqrt{\sum_{m=1}^N (\phi_i^m \cdot \Gamma_m)^2}$$

- distribution proportional to the masses (belonging to Group 2):

$$F_i = \frac{m_i}{\sum_{j=1}^N m_j} V_b$$

- adaptive load distribution (belonging to Group 2), continuously updated during the analysis, to reflect the progressive stiffness degradation of the structure. This distribution considers the contribution of multiple modes of vibration, updated at each load step (Adaptive Analysis module):

$$F_i = \sqrt{\sum_n \sum_m \rho_{n,m} \cdot F_{i,n} \cdot F_{i,m}}$$

where

$$F_{ij} = \Gamma_j \cdot \phi_{i,j} \cdot M_i \cdot S_a(T_j, \xi)$$

$$\rho_{n,m} = \frac{8 \xi^2 \beta_{i,j}^{3/2}}{(1 + \beta_{i,j}) \cdot [(1 - \beta_{i,j})^2 + 4 \cdot \xi^2 \cdot \beta_{i,j}]}$$

## LATERAL LOAD DISTRIBUTIONS ADOPTED IN EASYOVER

where

$$F_{ij} = \Gamma_j \cdot \phi_{i,j} \cdot M_i \cdot S_a(T_j, \xi)$$

$$\rho_{n,m} = \frac{8\xi^2 \beta_{i,j}^{3/2}}{(1 + \beta_{i,j}) \cdot [(1 - \beta_{i,j})^2 + 4 \cdot \xi^2 \cdot \beta_{i,j}]}$$

being

$i$  = node index;  $j, n, m$  = index of vibration mode

$\xi$  = damping factor for vibration modes

$\beta_{i,j} = T_j/T_i$  = inverse ratio between the periods of each  $i$ - $j$  pair of modes

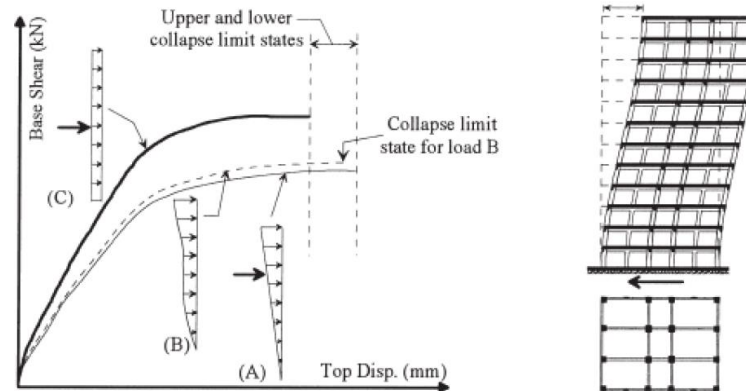
- adaptive load distribution (belonging to Group 2), continuously updated during the analysis, in equilibrium with the story shear distribution calculated by combining modal responses from a response spectrum analysis of the building (*Adaptive Analysis* module)

## LATERAL LOAD DISTRIBUTIONS ADOPTED IN EASYOVER

The resultant of the uniform distribution corresponds to the lowest point of application and therefore the maximum resistance and the lower yield and limit state of collapse displacements.

The resultant of the triangular distribution is applied to the highest point and presents, on the other hand, the lower resistance and the greater yield and the collapse limit state displacements.

The choice of considering two different distributions arises from the consideration that the distribution of the lateral forces should approximate the distribution of the forces of inertia during the earthquake. Comparisons with nonlinear dynamic analyses have shown that force distributions proportional to first vibrational mode better capture the dynamic response as long as the structure remains in the elastic field, whereas, when large deformations are reached, the dynamic response can be better represented by distributions of forces proportional to the masses.



## LATERAL LOAD DISTRIBUTIONS ADOPTED IN EASYOVER

In the adaptive analysis, the loads applied to the structure are updated at each stage, according to the secant stiffness of the elements (beam, plates and bricks) at the stage considered. The procedure can be summarized in the following steps:

- calculation of modal vectors:  $F_{ij} = \Gamma_j \cdot \phi_{i,j} \cdot M_i \cdot S_a(T_j, \xi)$

being

i = node index; j, n, m = index of vibration mode

$M_i$  mass of the i-th node of the structure

$\phi_{i,j}$  = displacement of the i-th node in the j-th mode

$\Gamma_j$  is the "modal participation factor" defined by the relation:

$$\Gamma_j = \frac{\phi_j^T M \tau}{\phi_j^T M \phi_j} \quad \text{where } \tau \text{ it is the drag vector corresponding to the direction of the considered seismic action}$$

$T_j$  period of the j-th vibration mode

$\xi$  = damping factor for vibration modes

$S_a$  = horizontal elastic acceleration response spectrum

- combination of modes and calculation of incremental load vector:

$$F_i = \sqrt{\sum_n \sum_m \rho_{n,m} \cdot F_{i,n} \cdot F_{i,m}}$$



## LATERAL LOAD DISTRIBUTIONS ADOPTED IN EASYOVER

where

$$\rho_{n,m} = \frac{8\xi^2 \beta_{i,j}^{3/2}}{(1 + \beta_{i,j}) \cdot [(1 - \beta_{i,j})^2 + 4 \cdot \xi^2 \cdot \beta_{i,j}]}$$

being

$i$  = node index;  $j, n, m$  = index of vibration mode

$\beta_{i,j} = T_j/T_i$  = inverse ratio between the periods of each  $i$ - $j$  pair of modes

- normalization of the incremental load vector:

$$\bar{F} = \frac{F}{\sum F_i}$$

- increase of the incremental load vector:

$$P_k = P_{k-1} + \Delta\lambda_k P_0 \bar{F}_k$$

being

$k$  = load step index

## LATERAL LOAD DISTRIBUTIONS ADOPTED IN EASYOVER

- adaptive load distribution (belonging to Group 2), continuously updated during the analysis, in equilibrium with the story shear distribution calculated by combining modal responses from a response spectrum analysis of the building (*Adaptive Analysis* module)

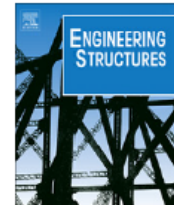
Engineering Structures 36 (2012) 160–172



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An adaptive modal pushover procedure for asymmetric-plan buildings

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## LATERAL LOAD DISTRIBUTIONS ADOPTED IN EASYOVER

The maximum induced modal forces and torques in each mode, because of excitation in one direction (the  $y$ -direction) is computed by Eqs. (11)–(13). These loads are applied in the mass center of the stories:

$$F_{x_{ij}} = \Gamma_{y_j} \phi_{x_{ij}} m_{x_i} S_{a_{yj}} \quad (11)$$

$$F_{y_{ij}} = \Gamma_{y_j} \phi_{y_{ij}} m_{y_i} S_{a_{yj}} \quad (12)$$

$$T_{\theta_{ij}} = \Gamma_{y_j} \phi_{\theta_{ij}} I_{\theta_i} S_{a_{yj}} \quad (13)$$

where  $i$  is the story number,  $j$  is the mode number,  $m_{x_i}$  is the translational mass of the  $i$ th story in  $x$  direction,  $m_{y_i}$  is the translational mass of the  $i$ th story in  $y$  direction,  $I_{\theta_i}$  is the rotational mass of the  $i$ th story and  $S_{a_{yj}}$  is the spectral acceleration in  $y$  direction corresponding to the  $j$ th mode.

In the well-known spectral-dynamic-analysis (SDA) the maximum total responses of the structure is approximated by the combination of responses due to maximum modal forces [Eqs. (11)–(13)]. Chopra and Goel [25] have extended this concept to nonlinear analysis by proposing the MPA procedure.

profile of the structure. The story drift of 3D structures consist of the translational and torsional relative displacements and these parameters are directly related to the story shears and total story torques, therefore in the proposed STA procedure the load vector associated with the combined modal story shear and torque profile is selected as the load pattern which would be the best representative of the damage in structures. The process of defining the load pattern at each step of the STA procedure is illustrated through a sample nine-story building in Fig. 1. This sample building is asymmetric about  $x$ -axis and  $y$ -axis (two horizontal directions) which is subjected to horizontal ground motion in one direction (the  $y$ -direction).

At each step, based on the instantaneous structural properties, the modal story forces in two translational (the  $x$  and  $y$ ) directions and modal torque applied in the mass centers of stories, are calculated for each considered modes by Eqs. (11)–(13) [Fig. 1(a)].

The story shears in translational directions and story torque associated with each mode are calculated by Eqs. (14)–(16) [Fig. 1(b), (c) and (d)]. Then the combined modal story shears and combined

## LATERAL LOAD DISTRIBUTIONS ADOPTED IN EASYOVER

modal story torques are calculated using the complete-quadratic-combination (CQC) or square-root-of-the-sum-of-the-squares (SRSS) rule [Eqs. (17)–(19), Fig. 1(e), (f) and (g)].

$$SS_{xij} = \sum_{k=i}^n F_{xkj} \quad (14)$$

$$\Rightarrow CSS_{x_i} = \sqrt{\sum_{j=1}^m SS_{xij}^2} \quad (17)$$

$$SS_{yij} = \sum_{k=i}^n F_{ykj} \quad (15)$$

$$\Rightarrow CSS_{y_i} = \sqrt{\sum_{j=1}^m SS_{yij}^2} \quad (18)$$

$$ST_{\theta ij} = \sum_{k=i}^n T_{\theta kj} \quad (16)$$

$$\Rightarrow CST_{\theta_i} = \sqrt{\sum_{j=1}^m ST_{\theta ij}^2} \quad (19)$$

where  $SS_{xij}$  and  $SS_{yij}$  are the story shears in floor  $i$  associated with mode  $j$  in  $x$  and  $y$  directions, respectively.  $ST_{\theta ij}$  is the story torque in floor  $i$  associated with mode  $j$ .  $CSS_{x_i}$  and  $CSS_{y_i}$  are the combined modal story shears in floor  $i$  associated with all the modes considered.  $CST_{\theta_i}$  is the combined modal story torque in floor  $i$  associated with all the modes considered.

The lateral forces (in two translational directions) and torques required to generate  $CSS_{x_i}$  and  $CSS_{y_i}$  (the combined modal story shears) and  $CST_{\theta_i}$  profiles are assumed as the load pattern. The required story forces in each direction and the required story torque are calculated by subtracting the combined modal story shears and combined modal story torques of consecutive stories using Eqs. (20)–(22) [Fig. 1(h), (i) and (j)]:

$$\begin{cases} F_{x_i} = CSS_{x_i} - CSS_{x_{i-1}} & i < n \\ F_{x_n} = CSS_{x_n} & i = n \end{cases} \quad (20)$$

$$\begin{cases} F_{y_i} = CSS_{y_i} - CSS_{y_{i-1}} & i < n \\ F_{y_n} = CSS_{y_n} & i = n \end{cases} \quad (21)$$

$$\begin{cases} T_{\theta_i} = CST_{\theta_i} - CST_{\theta_{i-1}} & i < n \\ T_{\theta_n} = CST_{\theta_n} & i = n \end{cases} \quad (22)$$

By applying these loads (Eqs. (20)–(22)) in the mass center of each story, the resulted story shear and torque would be equal to the combined modal story shears and torque in each story. Consequently, the

## LATERAL LOAD DISTRIBUTIONS ADOPTED IN EASYOVER

induced drift in each story would be equal to the combined modal story drift in linear-elastic structures. Therefore the authors believe that these loads (Eqs. (20)–(22)) could be an efficient load pattern in each step of pushover analysis of nonlinear structures.

The components of this load pattern are normalized with respect to the summation of the force components (base shear) in the earthquake excitation direction (the  $y$ -direction) using Eqs. (23)–(25). The incremental applied load profile at each step is computed by Eqs. (26)–(28):

$$\bar{F}_{x_i} = \frac{F_{x_i}}{\sum F_{y_i}} \quad (23)$$

$$\bar{F}_{y_i} = \frac{F_{y_i}}{\sum F_{y_i}} \quad (24)$$

$$\bar{T}_{\theta_i} = \frac{T_{\theta_i}}{\sum F_{y_i}} \quad (25)$$

$$\Delta \bar{F}_{x_i} = \Delta V_{b_y} \times \bar{F}_{x_i} \quad (26)$$

$$\Delta \bar{F}_{y_i} = \Delta V_{b_y} \times \bar{F}_{y_i} \quad (27)$$

$$\Delta \bar{T}_{\theta_i} = \Delta V_{b_y} \times \bar{T}_{\theta_i} \quad (28)$$

where  $\Delta V_{b_y}$  is the incremental amount of the base shear in the earthquake excitation direction (the  $y$ -direction).

Even though the quadratic combination rule (e.g. SRSS) is used in defining the load pattern, the calculated applied forces or torque in each story could be negative whenever the value of the combined modal shears or torque in one story is less than the one in the upper story [Fig. 1(e), (f) and (g)]. Therefore, this method is able to simulate the force or torque distribution profiles with reversal signs as observed in dynamic analysis [Fig. 1(h), (i) and (j)]. So the effects of the reversal signs in the higher modes are also simulated in the proposed load pattern.

LATERAL LOAD DISTRIBUTIONS ADOPTED IN EASYOVER

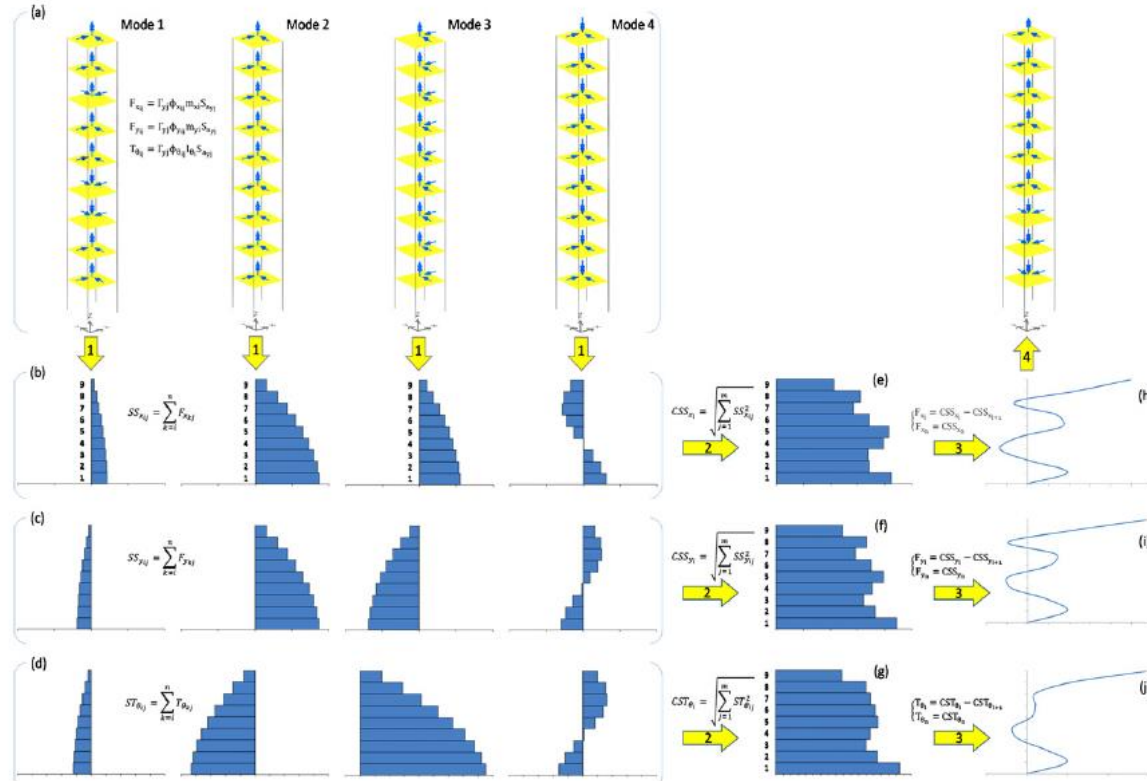


Fig. 1. The process of defining the load pattern at each step of the STA procedure. (a) Modal story forces in each mode. (b-d) Modal story shears and torques. (e-g) Combined modal story shear and torques. (h-j) Component of the proposed load pattern (x, y and  $\theta$  direction) in each story.

## LATERAL LOAD DISTRIBUTIONS ADOPTED IN EASYOVER

Definizione proprietà di calcolo - nomefile

Calcoli Push Over

Bilineare equivalente	Circ. n° 617 02/02/2009
Spinta prop. masse - dir. X	Solo Verso Positivo
Spinta modale - dir. X	No
Spinta multimodale - dir. X	No
Spinta prop. masse - dir. Y	No
Spinta modale - dir. Y	No
Spinta multimodale - dir. Y	No

Calcola Cerniere Plastiche    Forza laterale (% di  $m \cdot x \cdot g$ ): 2

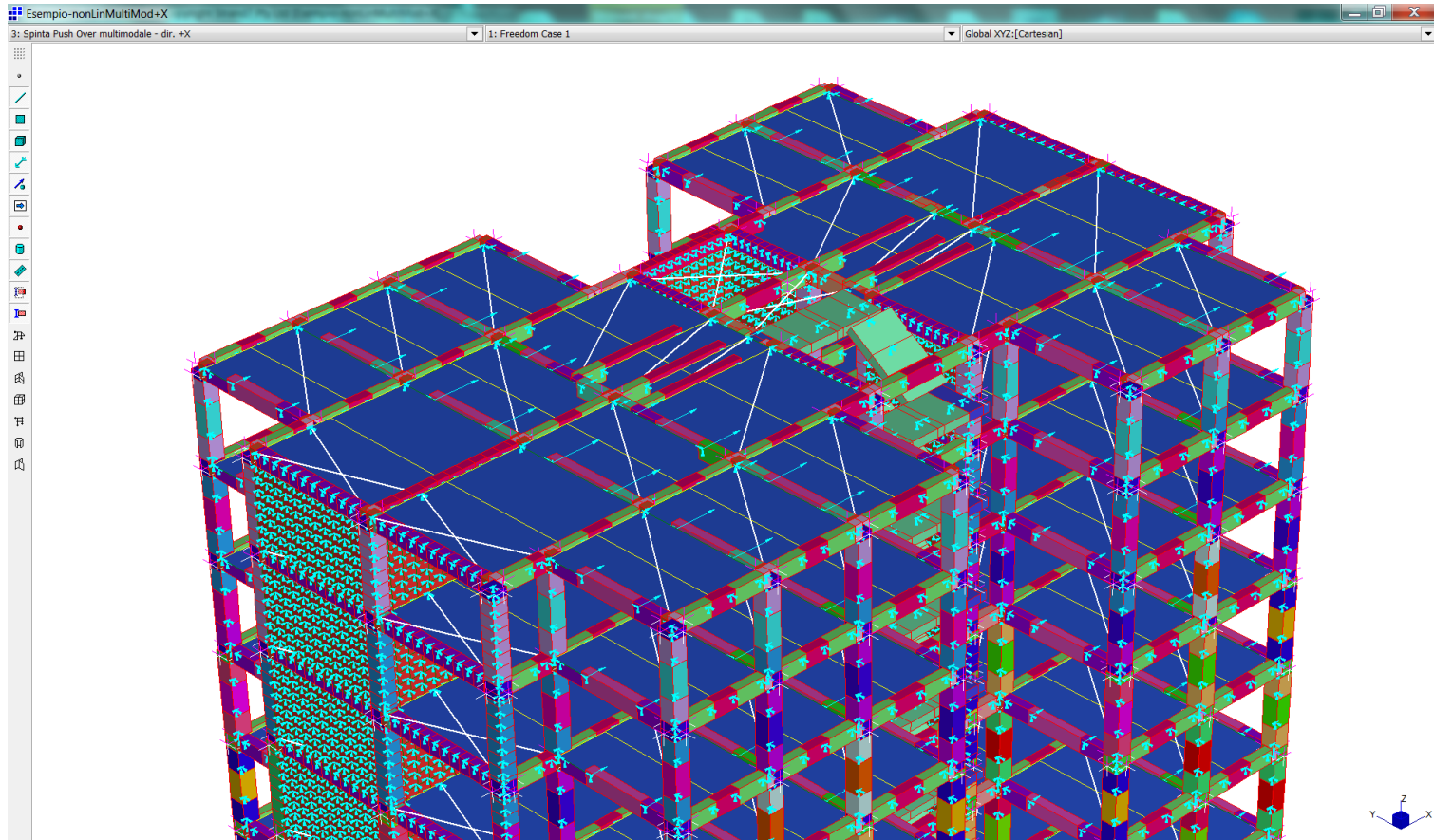
**Calcola Spinte**    Nodo di riferimento: 356

Analisi Non Lineare    Stato limite: DS

Calcola Domanda Sismica     Calcolo rapporto  $\alpha_u/\alpha_1$

Annulla

## LATERAL LOAD DISTRIBUTIONS ADOPTED IN EASYOVER





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